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Inflation with $\Omega \neq 1$

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Abstract

We discuss various models of inflationary universe with $\Omega \neq 1$. A homogeneous universe with $\Omega > 1$ may appear due to creation of the universe "from nothing" in the theories where the effective potential becomes very steep at large ϕ, or in the theories where the inflaton field ϕ nonminimally couples to gravity. Inflation with $\Omega < 1$ generally requires intermediate first order phase transition with the bubble formation, and with a second stage of inflation inside the bubble. It is possible to realize this scenario in the context of a theory of one scalar field, but typically it requires artificially bent effective potentials and/or nonminimal kinetic terms. It is much easier to obtain an open universe in the models involving two scalar fields. However, these models have their own specific problems. We propose three different models of this type which can describe an open homogeneous inflationary universe.

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1 Introduction

One of the most robust predictions of inflationary cosmology is that the universe after inflation becomes extremely flat, which corresponds to $\Omega = 1$. Here $\Omega = \frac{\rho}{\rho_c}$, ρ_c being the energy density of a flat universe. There were many good reasons to believe that this prediction was quite generic. The only way to avoid this conclusion is to assume that the universe inflated only by about e^{60} times. Exact value of the number of e-foldings N depends on details of the theory and may somewhat differ from 60. It is important, however, that in any particular theory inflation by an extra factor e^2 would make the universe with $\Omega = 0.5$ or with $\Omega = 1.5$ almost exactly flat. Meanwhile, the typical number of e-foldings, say, in chaotic inflation scenario in the theory $\frac{m^2}{2}\phi^2$ is not 60 but rather 10^{12} [1].

One can construct models where inflation leads to expansion of the universe by the factor e^{60}. However, in most of such models small number of e-foldings simultaneously implies that density perturbations are extremely large. Indeed, in most inflationary models the process of inflation begins at the point where density perturbations $\frac{\delta\rho}{\rho} \sim \frac{H^2}{\dot\phi}$ are very large. The simplest example is the original new inflation scenario [2], where inflation begins at the top of the effective potential with $\dot\phi = 0$. If there are less than 60 e-foldings from this moment to the end of inflation, then we would see extremely large density perturbations on the scale of horizon.

It may be possible to overcome this obstacle by a specific choice of the effective potential. However, this would be only a partial solution. Indeed, if the universe does not inflate long enough to become flat, then by the same token it does not inflate long enough to become homogeneous and isotropic. Thus, the main reason why it is difficult to construct inflationary models with $\Omega \neq 1$ is not the issue of fine tuning of the parameters of the models, which is necessary to obtain the universe inflating exactly e^{60} times, but the problem of obtaining a homogeneous universe after inflation.

Fortunately, it is possible to solve this problem, both for a closed universe [3, 4] and for an open one [4]–[9]. The main idea is to use the well known fact that the region of space created in the process of a quantum tunneling tends to have a spherically symmetric shape, and homogeneous interior, if the tunneling process is suppressed strongly enough. Then such bubbles of a new phase tend to evolve (expand) in a spherically symmetric fashion. Thus, if one could associate the whole visible part of the universe with an interior of one such region, one would solve the homogeneity problem, and then all other problems will be solved by the subsequent relatively short stage of inflation.

For a closed universe the realization of this program is relatively straightforward [3, 4]. One should consider the process of quantum creation of a closed inflationary universe from "nothing." If the probability of such a process is exponentially suppressed (and this is indeed the case if inflation is possible only at the energy density much smaller than the Planck density [12]), then

the universe created that way will be rather homogeneous from the very beginning.

The situation with an open universe is much more complicated. Indeed, an open universe is infinite, and it may seem impossible to create an infinite universe by a tunneling process. Fortunately, this is not the case: any bubble formed in the process of the false vacuum decay looks from inside like an infinite open universe [5]–[8]. If this universe continues inflating inside the bubble [7, 9, 4], then we obtain an open inflationary universe.

These possibilities became a subject of an active investigation only very recently, and there are still many questions to be addressed. First of all, the bubbles created by tunneling are not *absolutely* uniform even if the probability of tunneling is very small. This may easily spoil the whole scenario since in the end of the day we need to explain why the microwave background radiation is isotropic with an accuracy of about 10^{-5}. Previously we did not care much about initial homogeneities, but if the stage of inflation is short, we will the see original inhomogeneities imprinted in the perturbations of the microwave background radiation.

The second problem is to construct realistic inflationary models where all these ideas could be realized in a natural way. Whereas for the closed universe this problem can be easily solved [3, 4], for an open universe we again meet complications. It would be very nice to to obtain an open universe in a theory of just one scalar field [9]. However, in practice it is not very easy to obtain a satisfactory model of this type. Typically one is forced either to introduce very complicated effective potentials, or consider theories with nonminimal kinetic terms for the inflaton field [10]. This makes the models not only fine-tuned, but also rather complicated. It is very good to know that the models of such type in principle can be constructed, but it is also very tempting to find a more natural realization of the inflationary universe scenario which would give inflation with $\Omega < 1$.

This goal can be achieved if one considers models of two scalar fields [4]. One of them may be the standard inflaton field ϕ with a relatively small mass, another may be, e.g., the scalar field responsible for the symmetry breaking in GUTs. The presence of two scalar fields allows one to obtain the required bending of the inflaton potential by simply changing the definition of the inflaton field in the process of inflation. On the first stage the role of the inflaton is played by a heavy field with a steep barrier in its potential, while on the second stage the role of the inflaton is played by a light field, rolling in a flat direction "orthogonal" to the direction of quantum tunneling. This change of the direction of evolution in the space of scalar fields removes the naturalness constraints for the form of the potential, which are present in the case of one field.

Inflationary models of this type are quite simple, yet they have many interesting features. In these models the universe consists of infinitely many expanding bubbles immersed into exponentially expanding false vacuum state. Each of these bubbles inside looks like an open universe, but the values of Ω in these universes may take any value from 1 to 0. In some of these models the situation is even more complicated: Interior of each bubble looks like an infinite universe with

an effective value of Ω slowly decreasing to $\Omega = 0$ at an exponentially large distance from the center of the bubble. We will call such universes quasiopen. Thus, rather unexpectedly, we are obtaining a large variety of interesting and previously unexplored possibilities.

In this paper we will continue our discussion of inflationary models with $\Omega \neq 1$. In Section 2 we will describe a model of a closed inflationary universe. In Section 3 we will consider the possibility to implement an open inflation scenario in the theory of one scalar field. In Section 4 we discuss the issue of a spherical symmetry of the bubbles produced by a tunneling process. In Sections 5 – 7 we will describe several different models of an open inflationary universe. Finally, in the last Section of the paper we will summarize our results and discuss the most important question: What does inflationary cosmology say now to those trying to determine Ω from observational data?

2 Closed inflationary universe

For a long time it was not quite clear how can one obtain a homogeneous closed inflationary universe. In [9] it was even argued that this is impossible. Fortunately, it turns to be a relatively easy task [3, 4]. For example, one can consider a particular version of the chaotic inflation scenario [11] with the effective potential

$$V(\phi) = \frac{m^2\phi^2}{2}\, \exp\Big(\frac{\phi}{CM_{\rm P}}\Big)^2 . \tag{1}$$

Potentials of a similar type often appear in supergravity. In this theory inflation occurs only in the interval $\frac{M_{\rm P}}{2} \lesssim \phi \lesssim CM_{\rm P}$. The most natural way to realize inflationary scenario in this theory is to assume that the universe was created "from nothing" with the field ϕ in the interval $\frac{M_{\rm P}}{2} \lesssim \phi \lesssim CM_{\rm P}$. The universe at the moment of its creation has a size H^{-1}, and then it begins inflating as $H^{-1}\cosh Ht$. According to [12]–[15], the probability of creation of an inflationary universe is suppressed by

$$P \sim \exp\Big(-\frac{3M_{\rm P}^4}{8V(\phi)}\Big) . \tag{2}$$

Therefore the maximum of the probability appears near the upper range of values of the field ϕ for which inflation is possible, i.e. at $\phi_0 \sim CM_{\rm P}$ (see more discussion about this below). The probability of such an event will be so strongly suppressed that the universe will be formed almost ideally homogeneous and spherically symmetric. As pointed out in [3], this solves the homogeneity, isotropy and horizon problems even before inflation really takes over. Then the size of the newly born universe in this model expands by the factor $\exp(2\pi\phi_0^2 M_{\rm P}^{-2}) \sim \exp(2\pi C^2)$ during the stage of inflation [1]. If $C \gtrsim 3$, i.e. if $\phi_0 \gtrsim 3M_{\rm P} \sim 3.6 \times 10^{19}$ GeV, the universe expands more than e^{60} times, and it becomes very flat. Meanwhile, for $C \ll 3$ the universe always remains "underinflated" and very curved, with $\Omega \gg 1$. We emphasize again that in this

particular model "underinflation" does not lead to any problems with homogeneity and isotropy. The only problem with this model is that in order to obtain Ω in the interval between 1 and 2 at the present time one should have the constant C to be fixed somewhere near $C = 3$ with an accuracy of few percent. This is a fine-tuning, which does not sound very attractive. However, it is important to realize that we are not talking about an exponentially good precision; accuracy of few percent is good enough.

A similar result can be obtained even without changing the shape of the effective potential. It is enough to assume that the field ϕ has a nonminimal interaction with gravity of the form $-\frac{\xi}{2}R\phi^2$. In this case inflation becomes impossible for $\phi > \frac{M_{\rm P}}{\sqrt{8\pi\xi}}$ [16, 17]. Thus in order to ensure that only closed inflationary universes can be produced during the process of quantum creation of the universe in the theory $\frac{m^2}{2}\phi^2$ it is enough to assume that $\frac{M_{\rm P}}{\sqrt{8\pi\xi}} < 3M_{\rm P}$. This gives a condition $\xi > \frac{1}{72\pi} \sim 4 \times 10^{-4}$.

To make sure that this mechanism of a closed universe creation is viable one should check that the universe produced that way is sufficiently homogeneous. Even though one may expect that this is guaranteed by the large absolute value of the gravitational action, one should check that the asymmetry of the universe at the moment of its creation does not exceed an extremely small value $\sim 10^{-5}$, since otherwise our mechanism will produce anisotropy of the microwave background radiation exceeding its experimentally established value $\frac{\Delta T}{T} \sim 10^{-5}$.

Calculation of probability of creation of a closed universe is a very controversial subject, and determination of its quantum state and of its possible asymmetry is even more complicated. However, one can make a simple estimate and show that the absolute value of the action $|S| = \frac{3M_{\rm P}^4}{16V(\phi)}$ on the tunneling trajectory describing the universe formation will change by $\Delta|S| \sim O(1)$ if one adds perturbation of the standard amplitude $\delta\phi \sim \frac{H}{2\pi}$ to the homogeneous background of the scalar field ϕ. Trajectories with $\Delta|S| < 1$ are not strongly suppressed as compared with the original tunneling trajectory. Therefore tunneling into configurations with $\delta\phi \sim \frac{H}{2\pi}$ can be possible, whereas we expect that tunneling with creation of the universes with $\delta\phi \gg \frac{H}{2\pi}$ should be exponentially suppressed as compared with the tunneling with creation of the universes with $\delta\phi \sim \frac{H}{2\pi}$.

This result suggests that expected deviations of the scalar field from homogeneity at the moment of the universe creation have the usual quantum mechanical amplitude $\frac{H}{2\pi}$ which is responsible for galaxy formation and anisotropy of the microwave background radiation in the standard versions of inflation in a flat universe. In addition to this, there will be certain irregularities of the shape of the original bubble, since its size $\sim H^{-1}$ at the tunneling is defined with an accuracy $\sim M_{\rm P}^{-1}$. The resulting anisotropy $\sim \frac{H}{M_{\rm P}}$ is similar to the amplitude of gravitational waves produced during inflation. In other words, both scalar and gravitational perturbations induced at the moment of the universe creation are expected to be *of the same magnitude* as if the universe were inflating for an indefinitely long time. Therefore tunneling may play the same

role as inflation from the point of view of the homogeneity and isotropy problems [3, 4].

This possibility is very intriguing. Still, we do not want to insist that our conclusions are unambiguous. For example, one could argue that it is much more natural for the universe to be created with the density very close to the Planck density. However, the effective potential (1) at the Planck density is extremely steep. Therefore such a universe will not typically enter the inflationary regime, and will recollapse within a very short time. It could survive long enough for inflation to occur only if it was extremely large and relatively homogeneous from the very beginning. If the probability of creation of such a large universe is smaller than the probability of a direct creation of a homogeneous closed inflationary universe which we studied above, all our conclusions will remain intact. Some preliminary estimates of the probability of creation of a large universe which subsequently enters the stage of inflation can be found in [18]; however, this issue requires a more detailed investigation.

Leaving apart this cautious note, our main conclusion is that it may be possible to make inflation short and the universe closed and homogeneous. The remaining problem is to understand why our universe does not have $\Omega = 100$? But in fact it is very easy to answer this question: Value of Ω changes in a closed universe while it expands. It spends only a small fraction of its lifetime in a state with $\Omega \gg 1$. About a half of its lifetime before the closed universe becomes collapsing it has Ω only slightly greater than 1. Therefore a considerable fraction of all observers who may populate a closed universe should live there at the time when Ω is not much greater than 1. It is as simple as that. The situation with an open universe is much more complicated, since an open universe spends only a finite amount of time at the beginning of its evolution in a state with $\Omega \sim 1$, and then Ω decreases and stays for indefinitely long time in a state with $\Omega \ll 1$ ($\Omega \to 0$ for $t \to \infty$).

3 Open universe in the models with one scalar field

As we have already mentioned in the Introduction, the way to obtain an open homogeneous inflationary universe is to use the mechanism outlined in [5]–[9]. An open universe corresponds to an interior of a single bubble appearing in the decaying false vacuum. This picture can be consistent with observations only if the probability of tunneling with the bubble formation is sufficiently small, so that the bubbles do not collide until the typical size of the bubble becomes greater than the size of the observable part of our universe. The corresponding constraints are very easy to satisfy in the theories with small coupling constants, where the tunneling rate is very small [7]. However, if one modifies the theory in such a way that the probability of the bubble formation at some moment becomes so large that the phase transition completes in the whole universe (see e.g. [19]), then there will be a danger that an observer inside the bubble will see inhomogeneities created by other bubbles. Therefore will not study here theories of such type.

It is not very easy to find a reasonable model which will lead to tunneling and inflation by about e^{60} times after the bubble formation. The simplest idea [10] is to realize this scenario in the chaotic inflation model with the effective potential

$$V(\phi) = \frac{m^2}{2}\phi^2 - \frac{\delta}{3}\phi^3 + \frac{\lambda}{4}\phi^4 \ . \tag{3}$$

In order to obtain an open inflationary universe in this model it is necessary to adjust parameters in such a way as to ensure that the tunneling creates bubbles containing the field $\phi \sim 3M_{\rm P}$. In such a case the interior of the bubble after its formation inflates by about e^{60} times, and Ω at the present time may become equal to, say, 0.3. This requires a fine tuning of the effective potential. If, for example, tunneling occurs not to $\phi \sim 3M_{\rm P}$ but to $\phi \sim 3.1M_{\rm P}$ then the universe will become practically flat. It is worth noting, however, that the required fine tuning is about the same order as for the closed universe model described in Section 2, i.e. few percent.

Fine tuning is not the main difficulty of this model. The tunneling should occur to the part of the potential which almost does not change its slope during inflation at smaller ϕ, since otherwise one does not obtain scale-invariant density perturbations. One of the necessary conditions is that the barrier should be very narrow. Indeed, if $V'' \ll H^2$ at the barrier, then the tunneling occurs to its top, as in the Hawking-Moss case [20]. Original interpretation of this possibility was rather obscure; understanding came after the development of stochastic approach to inflation. What happens is that the inflaton field due to long-wave quantum fluctuations experiences Brownian motion. Occasionally this field may jump from the local minimum of the effective potential to its local maximum, and then slowly roll down from the maximum to the global minimum [21, 22, 3]. If this happens, one obtains unacceptably large density perturbations $\frac{\delta\rho}{\rho} \sim \frac{H^2}{\dot\phi} > 1$ on the large scale, since $\dot\phi = 0$ at the local maximum of $V(\phi)$.

Unfortunately, this is exactly the case for the model (3) [10]. Indeed, the local minimum of the effective potential in this model appears at

$$\phi = \frac{\delta}{2\lambda} + \gamma \ , \tag{4}$$

where

$$\gamma = \sqrt{\frac{\delta^2}{4\lambda^2} - \frac{m^2}{\lambda}} \ . \tag{5}$$

The local minimum of the effective potential appears for $\delta > 2\sqrt{\lambda}\, m$, and it becomes unacceptably deep (deeper than the minimum at $\phi = 0$) for $\delta > \frac{3\sqrt{\lambda}}{\sqrt{2}}\, m$. Thus in the whole region of interest one can use a simple estimate $\delta \sim 2\sqrt{\lambda}\, m$ and represent γ in the form $\beta\frac{m}{2\sqrt{2\lambda}}$, with $\beta < 1$. The local maximum of the potential appears at $\phi = \frac{\delta}{2\lambda} - \gamma$. Tunneling should occur to some point with $3M_{\rm P} < \phi < \frac{\delta}{2\lambda} - \gamma$, which implies that $\frac{\delta}{\lambda} > 6M_{\rm P}$.

The best way to study tunneling in this theory is to introduce the field χ in such a way that

$\chi = 0$ at the local minimum of $V(\phi)$:

$$\chi = -\phi + \frac{\delta}{2\lambda} + \gamma \ . \tag{6}$$

After simple algebra one can show that if the local minimum is not very deep ($\beta \ll 1$), the effective potential (3) can be represented as

$$V(\chi) \sim \frac{m^2\delta^2}{48\lambda^2} + \sqrt{2}\beta m^2\chi^2 - \frac{\delta}{3}\chi^3 + \frac{\lambda}{4}\chi^4 \ . \tag{7}$$

The Hubble constant in the local minimum is given by $H^2 \sim \frac{\pi\delta^2 m^2}{18\lambda^2 M_{\rm P}^2} > 2\pi\, m^2$, which is much greater than the effective mass squared of the field χ for $\beta \ll 1$, $m_\chi^2 = 2\sqrt{2}\beta m^2$. In this regime tunneling should occur to the local maximum of the effective potential, which should lead to disastrous consequences for the spectrum of density perturbations.

A possible way to overcome this problem is to consider the case $\delta \approx \frac{3\sqrt{\lambda}}{\sqrt{2}}\, m$ $(\beta \approx 1)$. Then the two minima of the effective potential become nearly degenerate in energy, and the Hubble constant becomes much smaller than m_χ. (In this regime our estimate for H, which was valid for $\beta \ll 1$, should be improved.) However, in such a situation we will have other problems. In this regime tunneling occurs almost exactly to the minimum of the effective potential at $\phi = 0$. Therefore it becomes difficult to have any inflation at all after the tunneling, and the problem of fine tuning becomes especially severe.

Note that this problem is rather general. Its origin is in the condition that in the inflationary universe scenario the curvature of the effective potential (the mass squared of the inflaton field) 60 e-foldings prior to the end of inflation always is much smaller than H^2. Therefore one should bend the effective potential in a quite dramatic way in order to create a local minimum at large ϕ and to make the curvature of the effective potential in this minimum much greater than H^2. One can avoid this problem by introducing non-minimal kinetic terms in the Lagrangian of the inflaton field [10], but this is just another representation of the artificial bending of the effective potential. Of course, it may happen that the bending of the effective potential can appear in a natural way in the theories based on supergravity and superstrings. The simplest idea is to multiply the effective potential (3) by the factor of the type $\exp\left(\frac{\phi}{CM_{\rm P}}\right)$ or $\exp\left(\frac{\phi}{CM_{\rm P}}\right)^2$, like in Eq. (1). At small ϕ these factors will not influence the shape of the effective potential, but at large ϕ they will make it very curved. This is exactly what we need to avoid the Hawking-Moss tunneling. Still the necessity to make all these tricks with bending the potential and making it very curved at some fine-tuned value of the scalar field ϕ do not make the models of this type particularly attractive.

Therefore in the next sections we will make an attempt to find some simple models where an open inflationary universe can appear in a more natural way. However, before doing so we will consider one more problem which should be addressed in all versions of the open inflationary universe scenario.

4 Tunneling probability and spherical symmetry

In the previous section we have assumed that the bubbles are exactly spherically symmetric (or, to be more accurate, $O(3,1)$-symmetric [5]). Meanwhile in realistic situations this condition may be violated for several reasons. First of all, the bubble may be formed not quite symmetric. Then its shape may change even further due to growth of its initial inhomogeneities and due to quantum fluctuations which appear during the bubble wall expansion. As we will see, this may cause a lot of problems if one wishes to maintain the degree of anisotropy of the microwave background radiation inside the bubble at the level of 10^{-5}.

First of all, let us consider the issue of symmetry of a bubble at the moment of its formation. For simplicity we will investigate the models where tunneling can be described in the thin wall approximation. We will neglect gravitational effects, which is possible as far as the initial radius r of the bubble is much smaller than H^{-1}. In this approximation (which works rather well for the models to be discussed) euclidean action of the $O(4)$-symmetric instanton describing bubble formation is given by

$$S = -\frac{\epsilon}{2}\pi^2 r^4 + 2\pi^2 r^3 s \ . \tag{8}$$

Here r is the radius of the bubble at the moment of its formation, ϵ is the difference of $V(\phi)$ between the false vacuum $\phi_{\rm initial}$ and the true vacuum $\phi_{\rm final}$, and s is the surface tension,

$$s = \int_{\phi_{\rm initial}}^{\phi_{\rm final}} \sqrt{2(V(\phi) - V(\phi_{\rm final}))}\, d\phi \ . \tag{9}$$

The radius of the bubble can be obtained from the extremum of (8) with respect to r:

$$r = \frac{3s}{\epsilon} \ . \tag{10}$$

Let us check how the action S will change if one consider a bubble of a radius $r + \Delta r$. Since the first derivative of S at its extremum vanishes, the change will be determined by its second derivative,

$$\Delta S = \frac{1}{2} S''(\Delta r)^2 = 9\pi^2\, \frac{s^2}{\epsilon}\, (\Delta r)^2 \ . \tag{11}$$

Now we should remember that all trajectories which have an action different from the action at extremum by no more than 1 are quite legitimate. Thus the typical deviation of the radius of the bubble from its classical value (10) can be estimated from the condition $\Delta S \sim 1$, which gives

$$|\Delta r| \sim \frac{\sqrt{\epsilon}}{3\pi\, s} \ . \tag{12}$$

Note, that even though we considered spherically symmetric perturbations, our estimate is based on corrections proportional to $(\delta r)^2$, and therefore it should remain valid for perturbations which have an amplitude Δr, but change their sign in different parts of the bubble surface. Thus,

eq. (12) gives an estimate of a typical degree of asymmetry of the bubble at the moment of its creation:

$$A(r) \equiv \frac{|\Delta r|}{r} \sim \frac{\epsilon\sqrt{\epsilon}}{3\pi\, s^2}\ . \tag{13}$$

This simple estimate exactly coincides with the corresponding result obtained by Garriga and Vilenkin [23] in their study of quantum fluctuations of bubble walls. It was shown in [23] that when an empty bubble begins expanding, the typical deviation Δr remains constant. Therefore the asymmetry given by the ratio $\frac{|\Delta r|}{r}$ gradually vanishes. This is a pretty general result: Waves produced by a brick falling to a pond do not have the shape of a brick, but gradually become circles.

However, in our case the situation is somewhat more complicated. The wavefront produced by a brick in inflationary background preserves the shape of the brick if its size is much greater than H^{-1}. Indeed, the wavefront moves with the speed approaching the speed of light, whereas the distance between different parts of a region with initial size greater than H^{-1} grows with a much greater (and ever increasing) speed. This means that inflation stretches the wavefront without changing its shape on scale much greater than H^{-1}. Therefore during inflation which continues inside the bubble the symmetrization of its shape occurs only in the very beginning, until the radius of the bubble approaches H^{-1}. At this first stage expansion of the bubble occurs mainly due to the motion of the walls rather than due to inflationary stretching of the universe, and our estimate of the bubble wall asymmetry as well as the results obtained by Garriga and Vilenkin for the empty bubble remain valid. At the moment when the radius of the bubble becomes equal to H^{-1} its asymmetry becomes

$$A(H^{-1}) \sim |\Delta r| H \sim \frac{\sqrt{\epsilon} H}{3\pi\, s}\ , \tag{14}$$

and the subsequent expansion of the bubble does not change this value very much. Note that the Hubble constant here is determined by the vacuum energy *after* the tunneling, which may differ from the initial energy density ϵ.

The deviation of the shape of the bubble from spherical symmetry implies that the beginning of the second stage of inflation inside the bubble will be not exactly synchronous, with the delay time $\Delta t \sim \Delta r$. This, as usual, may lead to adiabatic density perturbations on the horizon scale of the order of $H\Delta t$, which coincides with the bubble asymmetry A after its size becomes greater than H^{-1}, see Eq. (14).

To estimate this contribution to density perturbations, let us consider again the simplest model with the effective potential (3). Now we will consider it in the limit $\beta - 1 \ll 1$ which implies that the two minima have almost the same depth, which is necessary for validity of the thin wall approximation. In this case $2\delta^2 = 9M^2\lambda$, and the effective potential (3) looks approximately like $\frac{\lambda}{4}\phi^2(\phi-\phi_0)^2$, where $\phi_0 = \frac{2\delta}{3\lambda} = \sqrt{\frac{2}{\lambda}}M$ is the position of the local minimum of the effective potential. The surface tension in this model is given by $s = \sqrt{\frac{\lambda}{2}}\frac{\phi_0^3}{6} = \frac{M^3}{3\lambda}$ [24]. We will also

introduce a phenomenological parameter μ, such that $\mu\frac{M^4}{16\lambda} = \epsilon$. The smallness of this parameter controls applicability of the thin-wall approximation, since the value of the effective potential near the top of the potential barrier at $\phi = \phi_0/2$ is given by $\frac{M^4}{16\lambda}$. Then our estimate of density perturbations associated with the bubble wall (14) gives

$$\left.\frac{\delta\rho}{\rho}\right|_{\rm bubble} \sim A(H^{-1}) \sim \frac{\sqrt{\mu\lambda}H}{4\pi M} \ . \tag{15}$$

Here H is the value of the Hubble constant at the beginning of inflation inside the bubble.

In order to have $\left.\frac{\delta\rho}{\rho}\right|_{\rm bubble} \lesssim 5\times 10^{-5}$ (the number 5×10^{-5} corresponds to the amplitude of density perturbations in the COBE normalization) one should have

$$\left.\frac{\delta\rho}{\rho}\right|_{\rm bubble} \sim \frac{\sqrt{\mu\lambda}H}{4\pi M} \lesssim 5\times 10^{-5} \ . \tag{16}$$

For $H \ll M$ perturbations produced by the bubble walls may be sufficiently small even if the coupling constants are relatively large and the bubbles at the moment of their formation are very inhomogeneous.

There is a long way from our simple estimates to the full theory of anisotropies of cosmic microwave background induced by fluctuations of the domain wall. In particular, the significance of this effect will clearly depend on the value of Ω. The constraint (16) may appear only if one can "see" the scale at which the bubble walls have imprinted their fluctuations. If inflation is long enough, this scale becomes exponentially large, we do not see the fluctuations due to bubble walls, but then we return to the standard inflationary scenario of a flat inflationary universe. However, for $\Omega \ll 1$ inflation is short, and it does not preclude us from seeing perturbations in a vicinity of the bubble walls [25].[3] In such a case one should take the constraint (16) very seriously.

In the open universe the form of the spectrum of CMBR temperature fluctuations can be substantially different from the form of the spectrum of density fluctuations because of the integral Sachs-Wolfe effect [25] (see, in particular, the paper by Lyth and Woszczyna and references therein). In addition to this, the perturbations discussed above occur on a super-curvature scale. Therefore, they provide a natural source for the Grishchuk-Zeldovich effect (according to Lyth and Woszczyna [25] only the modes which are not in conformal vacuum can be responsible for this), while the density fluctuations produced during the secondary inflation are not likely to contain super-curvature modes.

One can show that in the theories of one scalar field similar to the model discussed in the previous section (if this model would work) the condition (16) is almost automatically satisfied. Meanwhile in the models of two different scalar fields, which we are going to discuss now, this condition may lead to additional restrictions on the parameters of the models.

[3] One should distinguish between the infinite size of an open universe and the finite distance from us to the bubble walls along the light cone.

5 The simplest model of a (quasi)open inflationary universe

As we have seen in Sect. 3, it is rather difficult to obtain an open universe in the models of one scalar field with simple potentials, such as, e.g., $\frac{m^2}{2}\phi^2 - \frac{\delta}{3}\phi^3 + \frac{\lambda}{4}\phi^4$. In this section we will explore an extremely simple model of two scalar fields, where the universe after inflation becomes open (or quasiopen, see below) in a very natural way [4].

Consider a model of two noninteracting scalar fields, ϕ and σ, with the effective potential

$$V(\phi, \sigma) = \frac{m^2}{2}\phi^2 + V(\sigma) \ . \tag{17}$$

Here ϕ is a weakly interacting inflaton field, and σ, for example, can be the field responsible for the symmetry breaking in GUTs. We will assume that $V(\sigma)$ has a local minimum at $\sigma = 0$, and a global minimum at $\sigma_0 \neq 0$, just as in the old inflationary theory. For definiteness, we will assume that this potential is given by $\frac{M^2}{2}\sigma^2 - \alpha M\sigma^3 + \frac{\lambda}{4}\sigma^4 + V(0)$, with $V(0) \sim \frac{M^4}{4\lambda}$, but it is not essential; no fine tuning of the shape of this potential will be required.

Note that so far we did not make any unreasonable complications to the standard chaotic inflation scenario; at large ϕ inflation is driven by the field ϕ, and the GUT potential is necessary in the theory anyway. In order to obtain density perturbations of the necessary amplitude the mass m of the scalar field ϕ should be of the order of $10^{-6}M_{\rm P} \sim 10^{13}$ GeV [1].

Inflation begins at $V(\phi, \sigma) \sim M_{\rm P}^4$. At this stage fluctuations of both fields are very strong, and the universe enters the stage of self-reproduction, which finishes for the field ϕ only when it becomes smaller than $M_{\rm P}\sqrt{\frac{M_{\rm P}}{m}}$ and the energy density drops down to $mM_{\rm P}^3 \sim 10^{-6}M_{\rm P}^4$ [1]. Quantum fluctuations of the field σ in some parts of the universe put it directly to the absolute minimum of $V(\sigma)$, but in some other parts the scalar field σ appears in the local minimum of $V(\sigma)$ at $\sigma = 0$. We will follow evolution of such domains. Since the energy density in such domains will be greater, their volume will grow with a greater speed, and therefore they will be especially important for us.

One may worry that all domains with $\sigma = 0$ will tunnel to the minimum of $V(\sigma)$ at the stage when the field ϕ was very large and quantum fluctuations of the both fields were large too. This may happen if the Hubble constant induced by the scalar field ϕ is much greater than the curvature of the potential $V(\sigma)$:

$$\frac{m\phi}{M_{\rm P}} \gtrsim M \ . \tag{18}$$

This decay can be easily suppressed if one introduces a small interaction $g^2\phi^2\sigma^2$ between these two fields, which stabilizes the state with $\sigma = 0$ at large ϕ. Another possibility, which we

have already mentioned in Sect. 2, is to add a nonminimal interaction with gravity of the form $-\frac{\xi}{2}R\phi^2$, which makes inflation impossible for $\phi > \frac{M_{\rm P}}{8\phi\xi}$. In this case the condition (18) will never be satisfied. However, there is a much simpler answer to this worry. If the effective potential of the field ϕ is so large that the field ϕ can easily jump to the true minimum of $V(\sigma)$, then the universe becomes divided into infinitely many domains with all possible values of σ distributed in the following way [21, 1]:

$$\frac{P(\sigma=0)}{P(\sigma=\sigma_0)} \sim \exp\left(\frac{3M_{\rm P}^4}{8V(\phi,0)} - \frac{3M_{\rm P}^4}{8V(\phi,\sigma)}\right) = \exp\left(\frac{3M_{\rm P}^4}{4(m^2\phi^2+2V(0))} - \frac{3M_{\rm P}^4}{4m^2\phi^2}\right) . \tag{19}$$

One can easily check that at the moment when the field ϕ decreases to $\frac{MM_{\rm P}}{m}$ and the condition (18) becomes violated, we will have

$$\frac{P(0)}{P(\sigma_0)} \sim \exp\left(-\frac{C}{\lambda}\right) , \tag{20}$$

where C is some constant, $C = O(1)$. After this moment the probability of the false vacuum decay typically becomes much smaller. Thus the fraction of space which survives in the false vacuum state $\sigma = 0$ until this time typically is very small, but finite (and calculable). It is important, that these rare domains with $\sigma = 0$ eventually will dominate the volume of the universe since if the probability of the false vacuum decay is small enough, the volume of the domains in the false vacuum will continue growing exponentially without end.

The main idea of our scenario can be explained as follows. Because the fields σ and ϕ do not interact with each other, and the dependence of the probability of tunneling on the vacuum energy at the GUT scale is negligibly small [5], tunneling to the minimum of $V(\sigma)$ may occur with approximately equal probability at all sufficiently small values of the field ϕ (see, however, below). The parameters of the bubbles of the field σ are determined by the mass scale M corresponding to the effective potential $V(\sigma)$. This mass scale in our model is much greater than m. Thus the duration of tunneling in the Euclidean "time" is much smaller than m^{-1}. Therefore the field ϕ practically does not change its value during the tunneling. If the probability of decay at a given ϕ is small enough, then it does not destroy the whole vacuum state $\sigma = 0$ [6]; the bubbles of the new phase are produced all the way when the field ϕ rolls down to $\phi = 0$. In this process the universe becomes filled with (nonoverlapping) bubbles immersed in the false vacuum state with $\sigma = 0$. Interior of each of these bubbles represents an open universe. However, these bubbles contain *different* values of the field ϕ, depending on the value of this field at the moment when the bubble formation occurred. If the field ϕ inside a bubble is smaller than $3M_{\rm P}$, then the universe inside this bubble will have a vanishingly small Ω, at the age 10^{10} years after the end of inflation it will be practically empty, and life of our type would not exist there. If the field ϕ is much greater than $3M_{\rm P}$, the universe inside the bubble will be almost exactly flat, $\Omega = 1$, as in the simplest version of the chaotic inflation scenario. It is important, however, that *in an eternally existing self-reproducing universe there will be infinitely many universes containing any particular value of* Ω, *from* $\Omega = 0$ *to* $\Omega = 1$, and one does not need any fine tuning of the effective potential to obtain a universe with, say, $0.2 < \Omega < 0.3$

Of course, one can argue that we did not solve the problem of fine tuning, we just transformed it into the fact that only a very small percentage of all universes will have $0.2 < \Omega < 0.3$. However, first of all, we achieved our goal in a very simple theory, which does not require any artificial potential bending and nonminimal kinetic terms. Then, there may be some reasons why it is preferable for us to live in a universe with a small (but not vanishingly small) Ω.

The simplest way to approach this problem is to find how the probability for the bubble productiondepends on ϕ. As we already pointed out, for small ϕ this dependence is not very strong. On the other hand, at large ϕ the probability rapidly grows and becomes quite large at $\phi > \frac{MM_{\rm P}}{m}$. This may suggest that the bubble production typically occurs at $\phi > \frac{MM_{\rm P}}{m}$, and then for $\frac{M}{m} \gg 3$ we typically obtain flat universes, $\Omega = 1$. This is another manifestation of the problem of premature decay of the state $\sigma = 0$ which we discussed above. Moreover, even if the probability to produce the universes with different ϕ were entirely ϕ-independent, one could argue that the main volume of the habitable parts of the universe is contained in the bubbles with $\Omega = 1$, since the interior of each such bubble inflated longer. Again, there exist several ways of resolving this problem: involving coupling $g^2\phi^2\sigma^2$, which stabilizes the state $\sigma = 0$ at large ϕ, or adding nonminimal interaction with gravity of the form $-\frac{\xi}{2}R\phi^2$, which makes inflation impossible for $\phi > \frac{M_{\rm P}}{\sqrt{8\pi\xi}}$. In either way one can easily suppress production of the universes with $\Omega = 1$. Then the maximum of probability will correspond to some value $\Omega < 1$, which can be made equal to any given number from 1 to 0 by changing the parameters g^2 and ξ. [4]

However, calculation of probabilities in the context of the theory of a self-reproducing universe is a very ambiguous process. For example, we may formulate the problem in a different way. Consider a domain of the false vacuum with $\sigma = 0$ and $\phi = \phi_1$. After some evolution it produces one or many bubbles with $\sigma = \sigma_0$ and the field ϕ which after some time becomes equal to ϕ_2. One may argue that the most efficient way this process may go is the way which in the end produces the greater volume. Indeed, for the inhabitants of a bubble it does not matter how much time did it take for this process to occur. The total number of observers produced by this process will depend on the total volume of the universe at the hypersurface of a given density, i.e. on the hypersurface of a given ϕ. If the domain instantaneously tunnels to the state σ_0 and ϕ_1, and then the field ϕ in this domain slowly rolls from ϕ_1 to ϕ_2, then the volume of this domain grows $\exp\Bigl(\frac{2\pi}{M_{\rm P}^2}(\phi_1^2 - \phi_2^2)\Bigr)$ times [1]. Meanwhile, if the tunneling takes a long time, then the field ϕ rolls down extremely slowly being in the false vacuum state with $\sigma = 0$. In this state the universe expands much faster than in the state with $\sigma = \sigma_0$. Since it expands much faster, and it takes the field much longer to roll from ϕ_1 to ϕ_2, the trajectories of this kind bring us much greater volume. This may serve as an argument that most of the volume is produced by the bubbles created at a very small ϕ, which leads to the universes with very small Ω.

One may use another set of considerations, studying all trajectories beginning at ϕ_1, t_1 and ending at ϕ_2, t_2. This will bring us another answer, or, to be more precise, another set of answers,

[4] Thus we disagree with the statement made in [19] that this model typically predicts empty universes.

which will depend on the choice of the time parametrization [26]. A very interesting approach was recently proposed by Vilenkin, who suggested to introduce a particular cutoff procedure which (almost) completely eliminates dependence of the final answer on the time parametrization [27]. A more radical possibility would be to integrate over all time parametrizations. This task is very complicated, but it would completely eliminate dependence of the final answer on the time parametrization [28].

There is a very deep reason why the calculation of the probability to obtain a universe with a given Ω is so ambiguous. For those who will live inside a bubble there will be no way to say at which stage (at which time from the point of view of an external observer) this bubble was produced. Therefore one should compare *all* of these bubbles produced at all possible times. The self-reproducing universe should exist for indefinitely long time, and therefore it should contain infinitely many bubbles with all possible values of Ω. Comparing infinities is a very ambiguous task, which gives results depending on the procedure of comparison. For example, one can consider an infinitely large box of apples and an infinitely large box of oranges. One may pick up one apple and one orange, then one apple and one orange, over and over again, and conclude that there is an equal number of apples and oranges. However, one may also pick up one apple and two oranges, and then one apple and two oranges again, and conclude that there is twice as many oranges as apples. The same situation happens when one tries to compare the number of bubbles with different values of Ω. If we would know how to solve the problem of measure in quantum cosmology, perhaps we would be able to obtain something similar to an open universe in the trivial $\lambda\phi^4$ theory without any first order phase transitions [28]. In the meantime, it is already encouraging that in our scenario there are infinitely many inflationary universes with all possible value of $\Omega < 1$. We can hardly live in the empty bubbles with $\Omega = 0$. As for the choice between the bubbles with different nonvanishing values of $\Omega < 1$, it is quite possible that eventually we will find out an unambiguous way of predicting the most probable value of Ω, and we are going to continue our work in this direction. However, it might also happen that this question is as meaningless as the question whether it is more probable to be born as a chinese rather than as an italian. It is quite conceivable that the only way to find out in which of the bubbles do we live is to make observations.

Some words of caution are in order here. The bubbles produced in our simple model are not *exactly* open universes. Indeed, in the models discussed in [5]–[9] the time of reheating (and the temperature of the universe after the reheating) was synchronized with the value of the scalar field inside the bubble. In our case the situation is very similar, but not exactly. Suppose that the Hubble constant induced by $V(0)$ is much greater than the Hubble constant related to the energy density of the scalar field ϕ. Then the speed of rolling of the scalar field ϕ sharply increases inside the bubble. Thus, in our case the field σ synchronizes the motion of the field ϕ, and then the hypersurface of a constant field ϕ determines the hypersurface of a constant temperature. In the models where the rolling of the field ϕ can occur only inside the bubble (we will discuss such a model shortly) the synchronization is precise, and everything goes as in the models of

refs. [5]–[9]. However, in our simple model the scalar field ϕ moves down outside the bubble as well, even though it does it very slowly. Thus, synchronization of motion of the fields σ and ϕ is not precise; hypersurface of a constant σ ceases to be a hypersurface of a constant density. For example, suppose that the field ϕ has taken some value ϕ_0 near the bubble wall when the bubble was just formed. Then the bubble expands, and during this time the field ϕ outside the wall decreases, as $\exp\left(-\frac{m^2 t}{3H_1}\right)$, where $H_1 \approx H(\phi = \sigma = 0)$ is the Hubble constant at the first stage of inflation, $H_1 \approx \sqrt{\frac{8\pi V(0)}{3M_P^2}}$ [1]. At the moment when the bubble expands e^{60} times, the field ϕ in the region just reached by the bubble wall decreases to $\phi_o \exp\left(-\frac{20m^2}{H_1^2}\right)$ from its original value ϕ_0. The universe inside the bubble is a homogeneous open universe only if this change is negligibly small. This may not be a real problem. Indeed, let us assume that $V(0) = \tilde{M}^4$, where $\tilde{M} = 10^{17}$ GeV. (Typically the energy density scale $\tilde{M}$ is related to the particle mass as follows: $\tilde{M} \sim \lambda^{-1/4} M$.) In this case $H_1 = 1.7 \times 10^{15}$ GeV, and for $m = 10^{13}$ GeV one obtains $\frac{20m^2}{H_1^2} \sim 10^{-4}$. In such a case a typical degree of distortion of the picture of a homogeneous open universe is very small.

Still this issue requires careful investigation. When the bubble wall continues expanding even further, the scalar field outside of it eventually drops down to zero. Then there will be no new matter created near the wall. Instead of infinitely large homogeneous open universes we are obtaining spherically symmetric islands of a size much greater than the size of the observable part of our universe. We do not know whether this unusual picture is an advantage or a disadvantage of our model. Is it possible to consider different parts of the same exponentially large island as domains of different "effective" Ω? Can we attribute some part of the dipole anisotropy of the microwave background radiation to the possibility that we live somewhere outside of the center of such island? In any case, as we already mentioned, in the limit $m^2 \ll H_1^2$ we do not expect that the small deviations of the geometry of space inside the bubble from the geometry of an open universe can do much harm to our model.

Our model admits many generalizations, and details of the scenario which we just discussed depend on the values of parameters. Let us forget for a moment about all complicated processes which occur when the field ϕ is rolling down to $\phi = 0$, since this part of the picture depends on the validity of our ideas about initial conditions. For example, there may be no self-reproduction of inflationary domains with large ϕ if one considers an effective potential of the field ϕ which is very curved at large ϕ, as in eq. (1). However, there will be self-reproduction of the universe in a state $\phi = \sigma = 0$, as in the old inflation scenario. Then the main portion of the volume of the universe will be determined by the processes which occur when the fields ϕ and σ stay at the local minimum of the effective potential, $\phi = \sigma = 0$. For definiteness we will assume here that $V(0) = \tilde{M}^4$, where $\tilde{M}$ is the stringy scale, $\tilde{M} \sim 10^{17} - 10^{18}$ GeV. Then the Hubble constant $H_1 = \sqrt{\frac{8\pi V(0)}{3M_P^2}} \sim \sqrt{\frac{8\pi}{3}}\frac{\tilde{M}^2}{M_P}$ created by the energy density $V(0)$ is much greater than $m \sim 10^{13}$ GeV. In such a case the scalar field ϕ will not stay exactly at $\phi = 0$. It will be relatively homogeneous on the horizon scale H_1^{-1}, but otherwise it will be chaotically distributed with the dispersion $\langle\phi^2\rangle = \frac{3H^4}{8\pi^2 m^2}$ [1]. This means that the field ϕ inside each of the bubbles produced by the decay

of the false vacuum can take any value ϕ with the probability

$$P \sim \exp\left(-\frac{\phi^2}{2\langle\phi^2\rangle}\right) \sim \exp\left(-\frac{3m^2\phi^2 M_{\rm P}^4}{16\tilde{M}^8}\right) \ . \tag{21}$$

One can check that for $\tilde{M} \sim 4.3 \times 10^{17}$ GeV the typical value of the field ϕ inside the bubbles will be $\sim 3 \times 10^{19}$ GeV. Thus, for $\tilde{M} > 4.3 \times 10^{17}$ GeV most of the universes produced during the vacuum decay will be flat, for $\tilde{M} < 4.3 \times 10^{17}$ GeV most of them will be open. It is interesting that in this version of our model the percentage of open universes is determined by the stringy scale (or by the GUT scale). However, since the process of bubble production in this scenario goes without end, the total number of universes with any particular value of $\Omega < 1$ will be infinitely large for any value of $\tilde{M}$. Thus this model shows us is the simplest way to resurrect some of the ideas of the old inflationary theory with the help of chaotic inflation, and simultaneously to obtain inflationary universe with $\Omega < 1$.

Note that this version of our model will not suffer for the problem of incomplete synchronization. Indeed, the average value of the field ϕ in the false vacuum outside the bubble will remain constant until the bubble triggers its decrease. However, this model, just as its previous version, may suffer from another problem. The Hubble constant H_1 before the tunneling in this model was much greater than the Hubble constant H_2 at the beginning of the second stage of inflation. Therefore the fluctuations of the scalar field before the tunneling were very large, $\delta\phi \sim \frac{H_1}{2\pi}$, much greater than the fluctuations generated after the tunneling, $\delta\phi \sim \frac{H_2}{2\pi}$. This may lead to very large density perturbations on the scale comparable to the size of the bubble. For the models with $\Omega = 1$ this effect would not cause any problems since such perturbations would be far away over the present particle horizon, but for small Ω this may lead to unacceptable anisotropy of the microwave background radiation.

Fortunately, this may not be a real difficulty. A possible solution is very similar to the bubble symmetrization described in the previous section.

Indeed, let us consider more carefully how the long wave perturbations produced outside the bubble may penetrate into it. At the moment when the bubble is formed, it has a size (5), which is smaller than H_1^{-1} [5]. Then the bubble walls begin moving with the speed gradually approaching the speed of light. At this stage the comoving size of the bubble (from the point of view of the original coordinate system in the false vacuum) grows like

$$r(t) = \int_0^t dte^{-H_1 t} = H_1^{-1}(1 - e^{-H_1 t}) \ . \tag{22}$$

During this time the fluctuations of the scalar field ϕ of the amplitude $\frac{H_1}{2\pi}$ and of the wavelength H_1^{-1}, which previously were outside the bubble, gradually become covered by it. When these perturbations are outside the bubble, inflation with the Hubble constant H_1 prevents them from oscillating and moving. However, once these perturbations penetrate inside the bubble, their

amplitude becomes decreasing [29, 30]. Indeed, since the wavelength of the perturbations is $\sim H_1^{-1} \ll H_2^{-1} \ll m^{-1}$, these perturbations move inside the bubbles as relativistic particles, their wavelength grow as $a(t)$, and their amplitude decreases just like an amplitude of electromagnetic field, $\delta\phi \sim a^{-1}(t)$, where a is the scale factor of the universe inside a bubble [29]. This process continues until the wavelength of each perturbation reaches H_2^{-1} (already at the second stage of inflation). During this time the wavelength grows $\frac{H_1}{H_2}$ times, and the amplitude decreases $\frac{H_2}{H_1}$ times, to become the standard amplitude of perturbations produced at the second stage of inflation: $\frac{H_2}{H_1}\frac{H_1}{2\pi} = \frac{H_2}{2\pi}$.

In fact, one may argue that this computation was too naive, and that these perturbations should be neglected altogether. Typically we treat long wave perturbations in inflationary universe like classical wave for the reason that the waves with the wavelength much greater than the horizon can be interpreted as states with extremely large occupation numbers [1]. However, when the new born perturbations (i.e. fluctuations which did not acquire an exponentially large wavelength yet) enter the bubble (i.e. under the horizon), they effectively return to the realm of quantum fluctuations again. Then one may argue that one should simply forget about the waves with the wavelengths small enough to fit into the bubble, and consider perturbations created at the second stage of inflation not as a result of stretching of these waves, but as a new process of creation of perturbations of an amplitude $\frac{H_2}{2\pi}$.

One may worry that perturbations which had wavelengths somewhat greater than H_1^{-1} at the moment of the bubble formation cannot completely penetrate into the bubble. If, for example, the field ϕ differs from some constant by $+\frac{H_1}{2\pi}$ at the distance H_1^{-1} to the left of the bubble at the moment of its formation, and by $-\frac{H_1}{2\pi}$ at the distance H_1^{-1} to the right of the bubble, then this difference remains frozen independently of all processes inside the bubble. This may suggest that there is some unavoidable asymmetry of the distribution of the field inside the bubble. However, the field inside the bubble will not be distributed like a straight line slowly rising from $-\frac{H_1}{2\pi}$ to $+\frac{H_1}{2\pi}$. Inside the bubble the field will be almost homogeneous; the inhomogeneity $\delta\phi \sim -\frac{H_1}{2\pi}$ will be concentrated only in a small vicinity near the bubble wall.

Finally we should verify that this scenario leads to bubbles which are symmetric enough, see eq. (16). Fortunately, here we do not have any problems. One can easily check that for our model with $m \sim 10^{13}$ GeV and $\tilde{M} \sim \lambda^{-1/4} M > 10^{17} GeV$ the condition (16) can be satisfied even for not very small values of the coupling constant λ.

The arguments presented above should be confirmed by a more detailed investigation of the vacuum structure inside the expanding bubble in our scenario. If, as we hope, the result of the investigation will be positive, we will have an extremely simple model of an open inflationary universe. In the meantime, it would be nice to have a model where we do not have any problems at all with synchronization and with large fluctuations on the scalar field in the false vacuum. We will consider such a model in the next section.

6 Hybrid inflation with $\Omega < 1$

The model to be discussed below [4] is a version of the hybrid inflation scenario [31], which is a slight generalization (and a simplification) of our previous model (17):

$$V(\phi, \sigma) = \frac{g^2}{2}\phi^2\sigma^2 + V(\sigma) \ . \tag{23}$$

We eliminated the massive term of the field ϕ and added explicitly the interaction $\frac{g^2}{2}\phi^2\sigma^2$, which, as we have mentioned already, can be useful (though not necessary) for stabilization of the state $\sigma = 0$ at large ϕ. Note that in this model the line $\sigma = 0$ is a flat direction in the (ϕ, σ) plane. At large ϕ the only minimum of the effective potential with respect to σ is at the line $\sigma = 0$. To give a particular example, one can take $V(\sigma) = \frac{M^2}{2}\sigma^2 - \alpha M\sigma^3 + \frac{\lambda}{4}\sigma^4 + V_0$. Here V_0 is a constant which is added to ensure that $V(\phi, \sigma) = 0$ at the absolute minimum of $V(\phi, \sigma)$. In this case the minimum of the potential $V(\phi, \sigma)$ at $\sigma \neq 0$ is deeper than the minimum at $\sigma = 0$ only for $\phi < \phi_c$, where $\phi_c = \frac{M}{g}\sqrt{\frac{2\alpha^2}{\lambda} - 1}$. This minimum for $\phi = \phi_c$ appears at $\sigma = \sigma_c = \frac{2\alpha M}{\lambda}$.

The bubble formation becomes possible only for $\phi < \phi_c$. After the tunneling the field ϕ acquires an effective mass $m = g\sigma$ and begins to move towards $\phi = 0$, which provides the mechanism for the second stage of inflation inside the bubble. In this scenario evolution of the scalar field ϕ is exactly synchronized with the evolution of the field σ, and the universe inside the bubble appears to be open.

Effective mass of the field ϕ at the minimum of $V(\phi, \sigma)$ with $\phi = \phi_c$, $\sigma = \sigma_c = \frac{2\alpha M}{\lambda}$ is $m = g\sigma_c = \frac{2g\alpha M}{\lambda}$. With a decrease of the field ϕ its effective mass at the minimum of $V(\phi, \sigma)$ will grow, but not significantly. For simplicity, we will consider the case $\lambda = \alpha^2$. In this case it can be shown that $V(0) = 2.77\,\frac{M^4}{\lambda}$, and the Hubble constant before the phase transition is given by $4.8\,\frac{M^2}{\sqrt{\lambda}M_p}$. One should check what is necessary to avoid too large density perturbations (16). However, one should take into account that the mass M in (16) corresponds to the curvature of the effective potential near $\phi = \phi_c$ rather than at $\phi = 0$. In our case this implies that one should use $\sqrt{2}M$ instead of M in this equation. Then one obtains the following constraint on the mass M: $M\sqrt{\mu} \lesssim 2 \times 10^{15}$ GeV. Note that the thin wall approximation (requiring $\mu \ll 1$) breaks down far away from $\phi = \phi_c$. Therefore in general eq. (16) should be somewhat improved. However for $\phi \approx \phi_c$ it works quite well. To be on a safe side, we will take $M = 5 \times 10^{14}$ GeV. Other parameters may vary; one may consider, e.g., the theory with $g \sim 10^{-5}$, which gives $\phi_c = \frac{M}{g} \sim 5 \times 10^{19}$ GeV $\sim 4M_{\rm P}$. The effective mass m after the phase transition is equal to $\frac{2gM}{\sqrt{\lambda}}$ at $\phi = \phi_c$, and then it grows by only 25% when the field ϕ changes all the way down from ϕ_c to $\phi = 0$. As we already mentioned, in order to obtain the proper amplitude of density perturbations produced by inflation inside the bubble one should have $m \sim 10^{13}$ GeV. This corresponds to $\lambda = \alpha^2 = 10^{-6}$.

The bubble formation becomes possible only for $\phi < \phi_c$. If it happens in the interval $4M_{\rm P} >$

$\phi > 3M_{\rm P}$, we obtain a flat universe. If it happens at $\phi < 3M_{\rm P}$, we obtain an open universe. Depending on the initial value of the field ϕ, we can obtain all possible values of Ω, from $\Omega = 1$ to $\Omega = 0$. The value of the Hubble constant at the minimum with $\sigma \neq 0$ at $\phi = 3M_{\rm P}$ in our model does not differ much from the value of the Hubble constant before the bubble formation. Therefore we do not expect any specific problems with the large scale density perturbations in this model. Note also that the probability of tunneling at large ϕ is very small since the depth of the minimum at $\phi \sim \phi_c$, $\sigma \sim \sigma_c$ does not differ much from the depth of the minimum at $\sigma = 0$, and there is no tunneling at all for $\phi > \phi_c$. Therefore the number of flat universes produced by this mechanism will be strongly suppressed as compared with the number of open universes, the degree of this suppression being very sensitive to the value of ϕ_c. Meanwhile, life of our type is impossible in empty universes with $\Omega \ll 1$. This may provide us with a tentative explanation of the small value of Ω in the context of our model (see, however, discussion of uncertainties related to this issue in Sect. 5).

7 "Supernatural" inflation with $\Omega < 1$

Natural inflation has been proposed some time ago [32, 33] as a model in which the inflaton field has a self-coupling constant whose smallness, required by the amplitude of cosmological inhomogeneities, is protected by the approximate global symmetry of the underlying particle physics. The pseudo-Nambu-Goldstone boson (PNGB) field ϕ, which serves as an inflaton in these models, would have been exactly massless if not for explicit $U(1)$ symmetry breaking induced by non-perturbative effects. The hierarchy between the scale of spontaneous symmetry breaking with generation of Nambu-Goldstone mode and the scale of explicit symmetry breaking which gives a mass to this mode is exploited to explain the smallness of the effective mass.

We will consider the case when the PNGB mode is described by the pseudo-scalar field ϕ, appearing as a phase of a complex scalar Φ. The scalar sector of the effective theory of the pseudo-Nambu-Goldstone mechanism is in general described by the action:

$$S(\Phi) = \int d^4x \sqrt{-g}\,(g^{\mu\nu}\partial_\mu\Phi^*\partial_\nu\Phi - V_0(|\Phi|) - V_1(\Phi)). \tag{24}$$

In notation $\Phi(x) = \frac{f(x)}{\sqrt{2}}\, e^{i\frac{\phi(x)}{f_0}}$ the field $f(x)$ is the radial component, the PNGB field $\phi(x)$ is the phase component, and f_0 is a dimensional parameter which is equal to the value of the scalar field $f(x)$ after symmetry breaking. The function $V_0(|\Phi|) = V_0(f)$ is the spontaneous symmetry breaking part of the potential for the complex scalar which remains globally $U(1)$ symmetric under the transformation $\phi(x) \to \phi(x) + c$. In the version of this theory considered in [33] this

potential was taken in the simplest form

$$V_0(f) = \frac{\lambda}{4}\left(f^2 - f_0^2\right)^2 \ . \tag{25}$$

The term $V_1(\Phi) = V_1(f,\phi)$ is the explicit $U(1)$ symmetry breaking potential which in many models takes the following form in the limit $f \to f_0$:

$$V_1(f,\phi) = \Lambda^4(f)\,\left(1 - \cos\Big(\frac{\phi}{f_0} - \bar{\theta}\Big)\right) . \tag{26}$$

Here $\Lambda^4(f)$ is some relatively slowly varying (in comparison with $V_0(f)$) function of the radial field, which vanishes at $f = 0$. This term may appear due to instantons in a theory with a gauge group with a "confinement" scale Λ, just like the term which is responsible for the axion mass. Another reason for appearance of such terms is the possibility that quantum gravity violates global symmetries [34]–[37]. This violation can be described by adding vertex operators of the type $\frac{g_{nm}}{M_{\rm P}^{m+n-4}}\,(\Phi^n\Phi^{*m}e^{-i\theta(n-m)} + h.c.)$. After spontaneous symmetry breaking the terms with the minimal degree of global symmetry violation ($|n-m| = 1$) lead to appearance of the terms of the type of (26).

The standard assumption of this scenario is that the effective potential $V_1(f,\phi)$ is much smaller than $V_0(f)$ everywhere except for $f \approx f_0$. Therefore the functional form of $\Lambda^4(f)$ should not be very important as far as (26) is the leading term in the PNGB effective potential at low energies, but does not give a significant contribution into the potential of the radial field away from the value f_0. Thus in what follows we will simply write $\Lambda \equiv \Lambda(f_0)$ instead of $\Lambda(f)$. Without loss of generality we can assume $\bar{\theta} = 0$ as long as there are no other terms in the low energy potential depending on it.

The potential in this theory resembles a slightly tilted mexican hat. In this scenario inflation occurs when the field ϕ falls from the maximum of the potential (26) at $\phi \sim \pi f_0$ (for $\bar{\theta} = 0$) to $\phi = 0$. Just like in the ordinary chaotic inflation scenario, the necessary condition for inflation to occur is $\pi f_0 \gtrsim M_{\rm P}$. Inflation will be long enough and density perturbations produced during this process will have sufficiently flat spectrum for $f_0 \gtrsim M_{\rm P}$. For definiteness, and in agreement with [33], we will assume here that $f_0 \sim M_{\rm P}$. The parameter Λ is determined by normalization of density perturbations produced during inflation. For $f_0 \sim M_{pl} \sim 10^{19}$ GeV one must have $\Lambda(f_0) \sim M_{GUT} \sim 10^{16}$ GeV (see [33] for detailed references).

In order for the radial part of the field to remain frozen and to not participate in natural inflation there has to be the case that $\lambda > \frac{32\pi}{3}\,\frac{\Lambda^4}{M_{\rm P}^2 f_0^2} \sim 10^{-12}$, which also ensures that the top of the "mexican hat" is higher than the highest energy of the axion Λ^4. The coupling constant λ could be as large as unity (however, it does not have to be so large). If we consider the energy density at the top of the "mexican hat" as the measure of how large is the coupling λ (we need

this characterization since we will soon change the shape of the potential), we see that there is a plenty of room for play — roughly 12 orders of magnitude in *energy density* between the GUT and Planck scales, or between the scales of spontaneous and explicit symmetry breaking.

The issue of naturalness of this scenario is not quite trivial. The first time the possibility of inflation of this kind was investigated by Binetruy and Gaillard in the context of superstring theory, and their conclusion was negative, for the reason that the typical value of the parameter f_0 in superstring theory is supposed to be few times smaller than $M_{\rm P}$ [32]. Still, the general idea of this scenario is rather elegant. Here we would like to suggest its generalization, which would allow us to obtain an inflationary universe with $\Omega < 1$. We will call our scenario "supernatural inflation" for its ability to accommodate naturally low Ω universes.[5]

Our main idea is to construct models which incorporate a primary stage of "old" inflation in the false vacuum state near $\Phi = 0$, and a first order transition which sets up for us the open de Sitter space as a stage for the subsequent secondary stage of inflation where the PNGB field ϕ plays the role of the inflaton. As before, if the number of e-foldings of secondary inflation will turn out to be just smaller 60, we will find ourselves in an open, yet matter-rich universe today.

In order to realize this scenario one should have a potential which has a minimum near $\Phi = 0$. A possible way to achieve it is to add to the Lagrangian the term $-g^2\chi^2\Phi^*\Phi$ describing interaction of the field Φ with some other scalar field χ. If the coupling constant of this interaction is sufficiently large ($g^4 > 16\pi^2\lambda$), the effective potential of the radial part f of the field Φ acquires a new minimum at $f = 0$ due to radiative corrections [1]. For $g^4 = 32\pi^2\lambda$ the minimum of the effective potential at $f = 0$ becomes as deep as the minimum at $f = f_0$, and the effective potential acquires the form $V(f) = \frac{\lambda}{2}(f_0^2 - f^2)f^2 + \lambda f^4 \ln\frac{f}{f_0}$. Thus for $16\pi^2\lambda < g^4 < 32\pi^2\lambda$ the potential has a minimum at $f = 0$ which is somewhat higher than the minimum at $f = f_0$. If g^4 is close to $16\pi^2\lambda$, the potential looks like the usual Coleman-Weinberg potential, and the phase transition from the state $f = 0$ occurs by the Hawking-Moss mechanism. However, one can show that if f_0 is not much greater than $M_{\rm P}$ and if g^4 does not differ much from $32\pi^2\lambda$ (which means that the minimum at $f = 0$ is deep enough), then the absolute value of the mass of the field f always remains greater than the Hubble constant. In such a situation the phase transition can be well described by the thin wall approximation neglecting gravitational effects, and (this is important) there will be no intermediate stage of inflation during the rolling of the field f towards f_0.

We do not want to discuss this issue here in great detail since radiative corrections is just one of the reasons why the effective potential may acquire a deep minimum at $f = 0$, and we would like to keep our discussion as general as possible. In particular, all subsequent results will remain valid if the potential $V(f)$ is given by the simple expression which we have used in the previous sections, $V(f) = \frac{m^2}{2}f^2 - \frac{\delta}{3}f^3 + \frac{\lambda}{4}f^4$. In order to increase the curvature of the effective potential at

[5]For the reason mentioned above, we do not want to imply any relation between this scenario and supersymmetric theories.

$\Phi = 0$ one may also add the term $\xi R\Phi^2$ to the Lagrangian. This term, being $U(1)$-invariant, does not affect the behavior of the Goldstone mode, and therefore it does not modify the standard picture of natural inflation, but it changes the curvature of the effective potential. It also may preclude inflation at large $f \gg f_0$. This may be useful, since otherwise inflation may begin at $f \gg f_0$, and then there will be no first order phase transition, and the universe will be flat. On the other hand, even if inflation begins at $f \gg f_0$, one still may experience the second stage of inflation at $\Phi = 0$ and the subsequent bubble formation if after the end of this stage of inflation at $f \gg f_0$ the oscillating scalar field has enough kinetic energy to climb to the local minimum of the effective potential at $\Phi = 0$.

An important feature of the "supernatural" inflation scenario is a very large difference between the energy density at the stage of inflation at $\Phi = 0$, which has the typical energy scale $\sim \frac{\lambda}{4} M_{\rm P}^4$, and the relatively small energy density Λ^4 during the last stage of inflation. As we have already mentioned, these two scales may differ from each other (for large λ) by about 12 orders of magnitude.

This implies that after the tunneling there will be a long intermediate stage of non-exponential expansion until the kinetic energy of the radial field and the energy density of particles produced by its oscillations becomes smaller than Λ^4. It takes time of the order of $H^{-1}(\Lambda) \sim M_{\rm P}/\Lambda^2$ to complete this intermediate stage. During the sub-luminal expansion epoch Ω stays very small and we can safely assume that the second inflationary stage starts at $\Omega = 0$. One can derive the adiabatic perturbation spectrum, modified by the fact that the natural inflation starts from the curvature dominated stage. Typically, the modification of the density perturbation spectrum is not great (remember, however, that the observable CMBR temperature fluctuations may differ considerably, see Section 4).

Note that in this scenario the Hubble constant during inflation at $f = 0$ is much greater than the Hubble constant at the second stage of inflation. In this respect supernatural inflation resembles the simple model which we discussed in Sect. 5. The main difference is that at the first stage of inflation in the supernatural scenario the mass of the field f is supposed to be much greater than the Hubble constant. (This condition is satisfied if the minimum at $f = 0$ is sufficiently deep.) Therefore there are no inflationary fluctuations of the scalar field produced outside the bubble in the supernatural inflation scenario. Moreover, even if the mass is not much greater than H, quantum fluctuations at $f = 0$ do not lead to any quantum fluctuations of the angular field ϕ, which is responsible for density perturbations after inflation. Thus, in this scenario we do not have any complications related to perturbations penetrating the bubble from the false vacuum.

In addition to the usual density perturbations produced during the second stage of inflation inside the bubble, there will be density perturbations induced by the initial inhomogeneities of the bubble. According to eq. (15), we can estimate the corresponding density perturbations as

follows:

$$\frac{\delta\rho}{\rho} \sim \frac{2\sqrt{2\mu}\Lambda^2}{\sqrt{3\pi} f_0 M_{\rm P}} \sim \frac{2\sqrt{2\mu}\Lambda^2}{\sqrt{3\pi} M_{\rm P}^2} \ , \tag{27}$$

where the last approximation assumes $f_0 \sim M_{\rm P}$. For $\Lambda \sim 10^{16}$ GeV and $\mu < 1$ these perturbations are smaller than the usual perturbations produced during the second stage of inflation.

If the energy in the maximum of the radial potential is much greater than the energy of the explicit symmetry breaking, the tunneling is likely to occur in any direction with equal probability. If it goes towards $\phi = 0$, one obtains an empty universe with $\Omega \ll 1$; if it goes towards $\phi \sim \pi f_0$, one obtains a universe with $\Omega \approx 1$. Thus we may say that it is about as likely to obtain $\Omega < 1$ as to obtain $\Omega = 1$, if we do not compare the volumes produced during the secondary inflation (the openness of the universes which we consider makes comparing the *a posteriori* volumes trickier). It is possible, though, to construct a (fine tuned) model which has a preferred value of Ω. To do this, we should discover a reason why the phase transition would go in a given preferred direction rather than any other.

So far we did not consider implications of the global symmetry breaking for the structure of the potential near $\Phi = 0$. If the corresponding terms appear due to instanton effects, which become significant only at the late stages of the universe evolution, the shape of the effective potential at small f remains unchanged. The physical reason is the infrared cutoff introduced by the Hawking temperature $T_H = \frac{H}{2\pi}$ near $\Phi = 0$. (The Hawking temperature may suppress effects induced by instantons if $H(\Phi = 0) \gg \Lambda$.) However, as we already mentioned, in addition to the low-energy non-perturbative effects, the high energy non-perturbative quantum gravitational effects may also add symmetry breaking terms of the form [34]–[37]

$$V_2(\phi(x)) = -\frac{1}{2} g_1\, M_{\rm P}^3\, (\Phi e^{-i\theta_1} + \Phi^* e^{i\theta_1}) = -g_1\, M_{\rm P}^3\, f(x) \cos\left(\frac{\phi(x)}{f_0} - \theta_1\right) . \tag{28}$$

The coupling g_1 may be strongly suppressed, so that (28) does not change the shape of the potential (26) at $f \sim f_0$ (see [37] for detailed explanations). However the linear term may play an important role at very small f, where all other terms are of a higher order in f. It may alter substantially the shape of the potential near the top of the mexican hat and determine the preferable direction of the tunneling. The phase θ_1, which determines the position of the minimum of the term (28), does not have to be the same as $\bar\theta$ from (26), so in general our potential acquires the form of a “twisted” mexican hat. If θ_1 happens to coincide with $\bar\theta$, then the tunneling typically will produce empty universes. If θ_1 differs from $\bar\theta$ by π, we will obtain the bubbles with typical values of $\Omega \sim 1$. For intermediate values of $\left|\theta_1 - \bar\theta\right|$ we will obtain predominantly $\Omega < 1$ universes. We should emphasize again that the calculation of probability of creation of universes with any particular value of Ω is not unambiguous, see Sect. 5. It is important that in all cases the total number of the universes with any possible value of Ω will be infinitely large.

8 Discussion

In this paper we suggested several different models of a homogeneous inflationary universe with $\Omega > 1$ and with $\Omega < 1$. At present there is no observational evidence in favor of $\Omega > 1$. It is clear, however, that if observational data are consistent with $\Omega = 1$, they may be consistent with $\Omega = 1.05$ as well. Therefore it is good to have a working model of inflationary cosmology with $\Omega > 1$.

The situation with an open universe may be much more interesting from the point of view of observational data. That is why in this paper we concentrated on the discussion of various models of inflationary universe with $\Omega < 1$.

We have found that in the models containing only one scalar field one typically needs fine tuning and rather artificial bending of the effective potential. In the models involving two scalar fields one typically obtains infinite number of open universes with all possible values of $\Omega < 1$ [4]. The simplest model of this type was discussed in Sect. 5. This model is very natural, but it has several unusual features. First of all, the universe described by this theory is not *exactly* homogeneous. A habitable part of it can be visualized as an exponentially large island inside an expanding bubble. The difference between the local properties of the island-like (quasiopen) universe and the homogeneous open inflationary universe in certain cases becomes very small. In these cases one should carefully analyse the fate of large perturbations of the scalar field which are generated outside the bubble but may penetrate inside it. We gave some arguments suggesting that these perturbations in this model may be harmless, but this question requires a more detailed investigation.

In Sect. 6 we proposed another model [4], which is based on a certain modification of the hybrid inflation scenario. This model is also very simple. It describe the universe which is open and homogeneous. Finally, in Sect. 7 we described a modified version of the natural inflation scenario. We do not want at this stage to discuss which of the open inflation models is better. It is clear that many other models of this type can be proposed. However, we think that there is a good chance that many of the qualitatively new features of the models discussed above will appear in the new models as well.

Should we take these models seriously? Should we admit that the standard prediction of inflationary theory that $\Omega = 1$ is not universally valid? We are afraid that now it is too late to discuss this question: the jinni is already out of the bottle. We know that inflationary models describing homogeneous inflationary universes with $\Omega \neq 1$ do exist, whether we like it or not. It is still true that the models which lead to $\Omega = 1$ are much more abundant and, arguably, more natural. However, in our opinion, it is very encouraging that inflationary theory is versatile enough to include models with all possible values of Ω.

To make our position more clear, we would like to discuss the history of the standard model of electroweak interactions [38]. Even though this model was developed by Glashow, Weinberg and Salam in the 60's, it became popular only in 1972, when it was realized that gauge theories with spontaneous symmetry breaking are renormalizable [39]. However, it was immediately pointed out that this model is far from being perfect. In particular, it was not based on the simple group of symmetries, and it had anomalies. Anomalies could destroy the renormalizability, and therefore it was necessary to invoke a mechanism of their cancellation by enlarging the fermion sector of the theory. This did not look very natural, and therefore Georgi and Glashow in 1972 suggested another model [40], which at the first glance looked much better. It was based on the simple group of symmetry $O(3)$, and it did not have any anomalies. In the beginning it seemed that this model is a sure winner. However, after the discovery of neutral currents which could not be described by the Georgi-Glashow model, everybody forgot about the issues of naturalness and simplicity and returned back to the more complicated Glashow-Weinberg-Salam model, which gradually became the standard model of electroweak interactions. This model has about twenty free parameters which so far did not find an adequate theoretical explanation. Some of these parameters may appear rather unnatural. The best example is the coupling constant of the electron to the Higgs field, which is 2×10^{-6}. It is a pretty unnatural number which is fine-tuned in such a way as to make the electron 2000 lighter than the proton. It is important, however, that all existing versions of the electroweak theory are based on two fundamental principles: gauge invariance and spontaneous symmetry breaking. As far as these principles hold, we can adjust our parameters and wait until they get their interpretation in a context of a more general theory. This is the standard way of development of the elementary particle physics.

For a long time cosmology developed in a somewhat different way, because of the scarcity of reliable observational data. Fifteen years ago many different cosmological models (HDM, CDM, $\Omega = 1$, $\Omega \ll 1$, etc.) could describe all observational data reasonably well. The main criterion for a good theory was its beauty and naturalness. Right now it becomes increasingly complicated to explain all observational data. Therefore cosmology is gradually becoming a normal experimental science, where the results of observations play a more important role than the considerations of naturalness. However, in our search for a correct theory we cannot give up the requirement of its internal consistency. In particle physics the two principles which made this theory internally consistent were the gauge invariance and spontaneous symmetry breaking. It seems that in cosmology something like inflation is needed to make the universe large and homogeneous. It is true that most of the inflationary models predict a universe with $\Omega = 1$. Hopefully, several years later we will know that our universe is flat, which will be a strong experimental evidence in favor of inflationary cosmology in its simplest form. However, if observational data will show, beyond any reasonable doubt, that $\Omega \neq 1$, it will not imply that inflationary theory is wrong, just like the discovery of neutral currents did not disprove gauge theories of electroweak interactions. Indeed, now we know that there is a large class of internally consistent cosmological models which may describe creation of large homogeneous universes with all possible values of Ω, and so far all of these models are based on inflationary cosmology.

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Figure Captions

Fig. 1 Effective potential $V(\phi, \sigma) = \frac{m^2}{2}\phi^2 + V(\sigma)$, eq. (17). Arrows show evolution of the fields σ and ϕ. Dashed lines correspond to tunneling, whereas the solid lines show slow rolling.

Fig. 2 Effective potential $V(\phi, \sigma) = \frac{g^2}{2}\phi^2\sigma^2 + V(\sigma)$, eq. (23).

Fig. 3 Effective potential in the model of supernatural inflation.